



The Mathematics of Voting: How to Upgrade Our Democracy

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A Brief Look at Democracy

“It has been said that democracy is the worst form of Government except for all those other forms that have been tried.”

- Winston Churchill

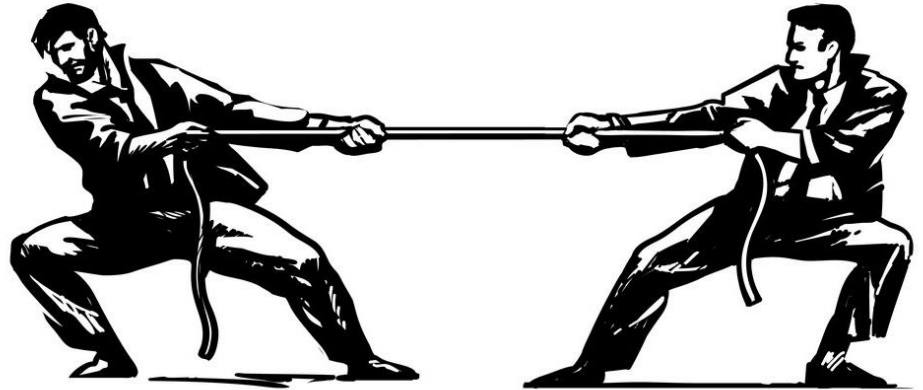


Figure 1: Anarchy vs. Tyranny



A Brief Look at Democracy II

It has been argued that, by many metrics, democracy contributes to human flourishing (Pinker, 2018).

Democracy has expanded worldwide, particularly post WWII. (Huntington, 1991).

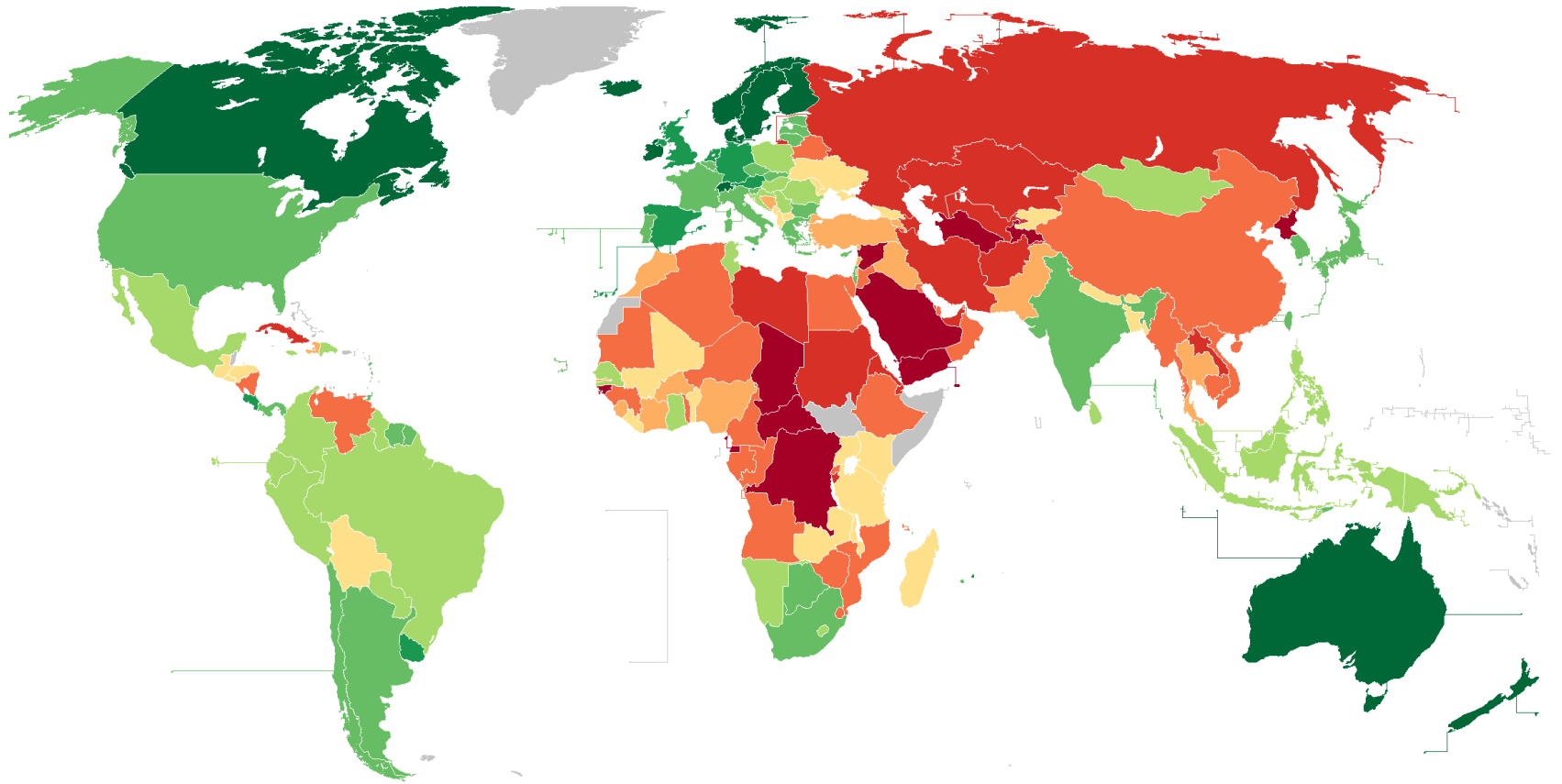


Figure 2: Democracy Worldwide: Greener is More Democratic

1
1 Economist Intelligence Unit: Democracy Index Map for 2018

The Worst Form of Democracy

1. What is the purpose of voting?
 2. What type of voting methods do most democracies use?
 3. How can we improve our election procedures?
 4. Does maximizing the voter's freedom of expression also maximize the accuracy of the societal election outcome?
- Not necessarily!**

Let's Just Overthrow the Electoral College...Or Not

Would replacing the electoral college with a popular vote work?

- Elimination requires a *Constitutional amendment*...
- ratified by two-thirds of the House and Senate...
- and by three-fourths of the states.



Method 1: Positional Voting Methods

Given, say, five candidates, associate a voting vector $(w_1, w_2, w_3, w_4, w_5)$ where the ballots are tallied by assigning w_j points (for $j= 1, \dots, 5$) to a voter's j th ranked candidate. The candidate with the most points wins.

- The Plurality Vote: $(1, 0, 0, 0, 0)$
- The Antiplurality Vote: $(1, 1, 1, 1, 0)$
- The Borda Count: $(4, 3, 2, 1, 0)$

Method 2: The Condorcet Method

Compare candidates in pairwise majority votes. This is a highly regarded voting method. What are some downfalls of this approach?

- The Enormous calculations.
- Cyclic preferences.

Most preferred \rightarrow *Least preferred.*

Voter 1 prefers $A > B > C$

Voter 2 prefers $B > C > A$

Voter 3 prefers $C > A > B$



Figure 3: Rocks, papers, scissors.

Method 3: Instant Runoffs or How I Was Rejected by Harvard University.

Applicants A through H apply for the position. An instant runoff pairwise comparisons is used to determine the new faculty position.

Most preferred → *Least preferred.*

Voter 1 prefers $A > B > C > D > E > F > G > H$

Voter 2 prefers $B > C > D > E > F > G > H > A$

Voter 3 prefers $C > D > E > F > G > H > A > B$



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	<i>Most preferred</i>	→	<i>Least preferred.</i>
Voter 1 prefers	A > B > C > D >		> H
Voter 2 prefers	B > C > D >		> H > A
Voter 3 prefers	C > D >		> H > A > B

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Voter 1 prefers	A > B >		> H
Voter 2 prefers	B >		> H > A
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Applicants A through H apply for the position. An instant runoff pairwise comparisons is used to determine the new faculty position.

Most preferred → *Least preferred.*

Voter 1 prefers		H
Voter 2 prefers		H
Voter 3 prefers	H	

| H is the winner. What a strong candidate H is!

How to Win Best Picture at the Oscars

- Voters rank all nominees.
An outright win is an outright win.
- Otherwise, the likely scenario, there's a runoff.
- The film with the fewest first-place votes is eliminated. Rinse and repeat until one film reaches an outright win, at least 50% first-place choices.



Good Film Hunting

Consider films ranked in the following order. Who wins best picture?

A > C > B

A > C > B

A > C > B

C > A > B

C > A > B

B > C > A

B > C > A

B > C > A

B > C > A

Good Film Hunting

Consider films ranked in the following order. Who wins best picture?

A > B

A > B

A > B

A > B

A > B

B > A

B > A

B > A

B > A

Good Film Hunting

Consider films ranked in the following order. Who wins best picture?

$A > C > B$

$A > C > B$

$A > C > B$

$C > A > B$

$C > A > B$

$B > C > A$

$B > C > A$

$B > C > A$

$B > C > A$

Which film is the plurality winner? The Condorcet winner?

Who 'Really' Won?

“Rather than reflecting the views of the voters, it is entirely possible for an election outcome to more accurately reflect the choice of an election procedure.” - Donald Saari

Isn't There a Big Theorem Telling Us to Give Up?

Assume the following:

- Voter's preferences are transitive with no restrictions on how they can rank the candidates.
- Societal rankings are transitive (ties are permitted).
- The Pareto condition holds. Namely, if all voters prefer A to B, then the election outcome should rank A above B.
- The Independence from Irrelevant Alternatives (IIA) holds. That is, if A ranks above B, then removing a third candidate C, should not change the ranking of A above B.

Theorem (Arrow's Impossibility Theorem)

For an election with three or more candidates, the only election procedure that satisfies the above conditions is a dictatorship.

Dear Mathematics, Which Voting Method is Best?

Of the positional voting methods, which method is

Q1: The most likely to agree with the Condorcet winner?

A1: When there is a Condorcet winner, it will agree with the Borda count winner. The Borda count is the unique positional voting method to guarantee this (Saari).

Q2: The most resistant to tactical voting?

A2: For a 3-way race, the Borda count is the most resistant to tactical voting, the plurality and antiplurality votes are the least resistant (Saari).

“It has been said that the Borda count is the worst form of Democracy except for all those other forms that have been tried.”
- Springsteen Kirkspill

Thank you!
www.michaelrpilla.com

Thanks for Coming!

We'll see you **Wednesday, April 15** for
“Better Science Through Beer” with
Kevin Smolar, 2014 M.S. Graduate of
Forensic Science

Learn more and register:
Science.IUPUI.edu/events



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