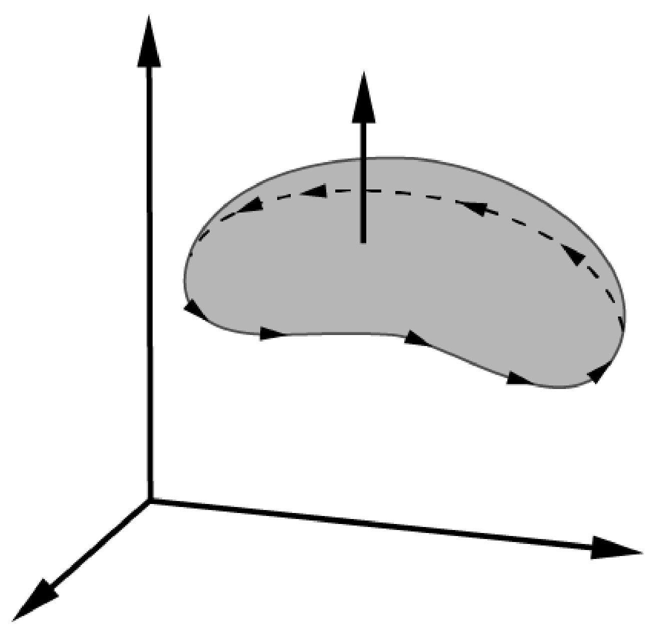
16.8: Stokes' Theorem.

OBJECTIVE

- Introduce Stokes' Theorem.

Recall that Green's theorem related a line integral over a planar curve with a double integral over some region. Stokes' theorem is a generalization in which we relate a line integral over a *space curve* to a surface integral.



Stokes' Theorem: Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation. Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives on an open region in R^3 that contains S. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

Note that S can be any surface as long as it has boundary curve given by C.

Also recall that we were able to write Green's theorem in vector form as

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{k}\, dA.$$

How does this relate to the above equation? (hint: recall that $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{n}$ is a unit normal vector to S.)

Example: Use Stokes' theorem to evaluate $\iint_S \text{curl}\,\mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = z^2\,\mathbf{i} - 3xy\,\mathbf{j} + x^3y^3\,\mathbf{k}$ and S is the part of $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume that S is oriented upwards.

Example: Use Stokes' theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = z^2\,\mathbf{i} + y^2\,\mathbf{j} + x\,\mathbf{k}$ and C is the triangle with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$. Assume counter-clockwise rotation.

Homework:

- Section 16.8: 1-9 odds