16.1: Vector Fields.

OBJECTIVE

- Introduce Vector Fields.
- Introduce the Change of Variables Equations.

We began with a definition.

Definition: A *vector field* on two [or three] dimensional space is a function $\mathbf{F}$ that assigns each point (x, y) [or (x, y, z)] a two [or three] dimensional vector given by $\mathbf{F}(x, y)$ [or $\mathbf{F}(x, y, z)$].

We often express this as

$$\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$$

for two dimensions and

$$\mathbf{F}(x, y, z) = \langle P(x, y, z), Q(x, y, z), R(x, y, z) \rangle$$
$$= P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$$

where the component functions P, Q and R are called the scalar functions.

Example: Sketch the vector field $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$.

An important vector field that we have already seen is the gradient vector field.

The **gradient vector field** is given by

$$\nabla f = \langle f_x, f_y \rangle \quad \text{or} \quad \nabla f = \langle f_x, f_y, f_z \rangle$$

Example: Find the gradient vector field of $f(x, y, z) = ye^{-xz}$.

Example: Sketch the gradient vector field of $f(x, y) = x^2 + y^2$ as well as several contours (level curves) for this function. How are they related?

Definition: A vector field $\mathbf{F}$ is called a **conservative vector field** if it is the gradient of some scalar function, that is, if there exists a function f such that $\mathbf{F} = \nabla f$. We call f a **potential function** for $\mathbf{F}$.

Homework:

- Section 16.1: 1-17, 21-25, 29-31 odds