15.5: Surface Area.

OBJECTIVE

- Find the Surface Area of a general region.

In previous calculus courses you may have seen surface areas by way of surfaces of revolution. Here we will determine the surface area of a region in a much more general setting.

We would like to know the surface area of a surface given by $z = f(x, y)$ where (x, y) is in our domain D in the xy-plane. Presuming that f_x and f_y are continuous, we find the following formula:

$$A(S) = \iint_D \sqrt{{f_x}^2 + {f_y}^2 + 1}\, dA$$

It can be shown (see Section 16.6) that this formula agrees with the formula for the area of a surface of revolution. Alternatively, we may write

$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

Example: Find the surface area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

Example: Find the surface area of the part of $z = xy$ that lies in the cylinder $x^2 + y^2 = 1$.

Homework:

- Section 15.5: 1-11 odds