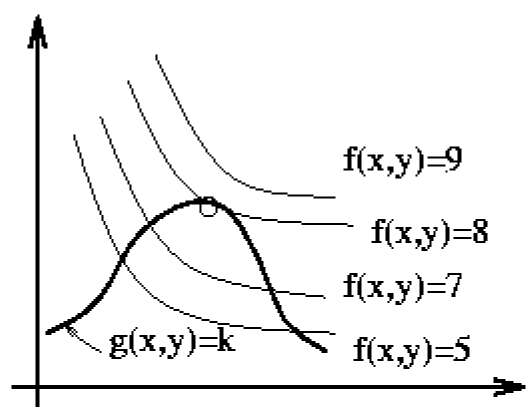
14.8: Lagrange Multipliers.

OBJECTIVE

- Introduce Lagrange's method of max/mins for a function subject to constraints.

Our goal is to maximize a function $f(x, y, z)$ subject to a constraint $g(x, y, z) = k$. We will start in two variables. We are looking for extreme values of $f(x, y)$ subject to the constraint $g(x, y) = k$.



When does this happen?

Method of Lagrange Multipliers: To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$

a. Find all values x, y, z and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) \quad \text{and} \quad g(x, y, z) = k.$$

b. Evaluate f at all the points (x, y, z) that result from above. The largest is the maximum of f; the smallest is the minimum of f.

What does this look like in terms of vector components?

Example: Find the dimensions of the box with largest volume if the total surface area is 64 cm^2.

Example: Find the maximum and minimum of $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$.

Two Constraints: Suppose we now would like to find the maximum and minimum values of $f(x, y, z)$ subject to two constraints $g(x, y, z) = k$ and $h(x, y, z) = c$. If f has an extreme value at (x_0, y_0, z_0) Then, presuming $\nabla g(x_0, y_0, z_0)$ and $\nabla h(x_0, y_0, z_0)$ are not zero and not parallel, we can show there are numbers λ and μ, called Lagrange multipliers, such that

$$\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0) + \mu \nabla h(x_0, y_0, z_0).$$

Example: Find the maximum and minimum distance from the origin to the curve of intersection of the surfaces $z^2 = x^2 + y^2$ subject to the constraints $x - 2z = 3$.

Homework:

- Section 14.8: 3-23