14.7: Maximum and Minimum Values.

OBJECTIVE

- Find local and absolute max/min values in two variables.

Recall that one of the most important applications of calculus in one variable was to find extreme (max/min) values. In this section we will focus on find extreme values of functions of two variables using partial derivatives.

Definition: A function of two variables has a **local maximum** at (a, b) if $f(x, y) \leq f(a, b)$ when (x, y) is near (a, b). We say $f(a, b)$ is the **local maximum value**. A function of two variables has a **local minimum** at (a, b) if $f(x, y) \geq f(a, b)$ when (x, y) is near (a, b). We say $f(a, b)$ is the **local minimum value**.

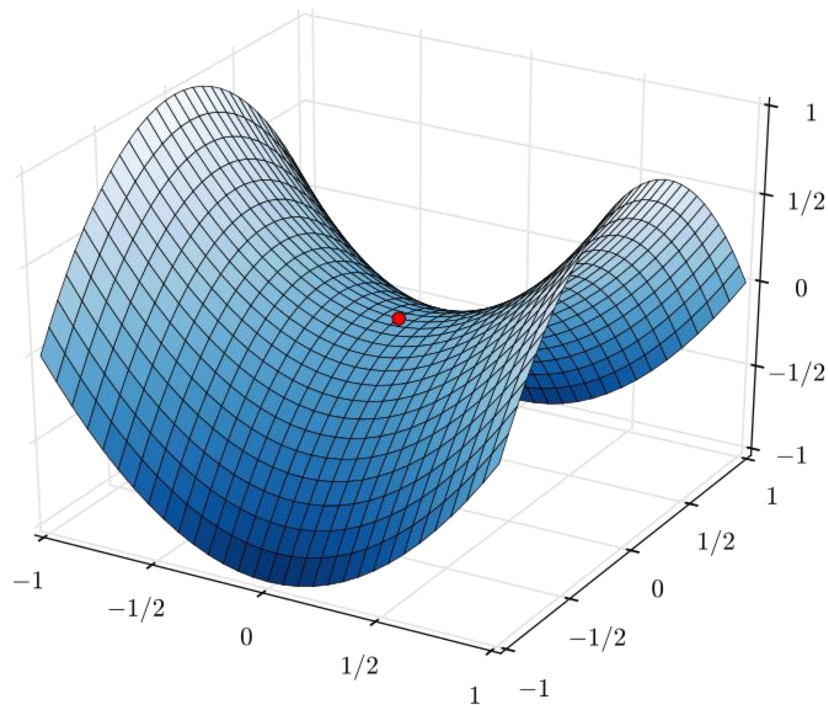
If the above inequalities hold for all points in our domain, we say f has an **absolute maximum** or **absolute minimum** at (a, b).

Theorem: If f is has a local maximum or minimum at (a, b) and the first-order partial derivatives of f exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

If $f_x(a, b) = 0$ and $f_y(a, b) = 0$ (or if one of the partial derivatives does not exist) we call (a, b) a **critical point**. If we insert these in the equation of a tangent plane, what does that tell us?

As in one variable, not all critical points give extreme values. Remember, all extrema are critical points, but not all critical points are extrema.

Example: Find the extreme values of $z = f(x, y) = y^2 - x^2$.



Second Derivative Test: Suppose the second partial derivatives of f are continuous on a disk with center (a, b) and suppose that $f_x(a, b) = 0$ and and $f_y(a, b) = 0$. Let

$$\mathrm{D} = \mathrm{D}(\mathrm{a}, \mathrm{b}) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

a) If $\mathrm{D} > 0$ and $f_{xx}(a, b) > 0$ then $f(a, b)$ is a local minimum.
b) If $\mathrm{D} > 0$ and $f_{xx}(a, b) < 0$ then $f(a, b)$ is a local maximum.
c) If $\mathrm{D} < 0$ then $f(a, b)$ is not a local minimum or maximum.
d) If $\mathrm{D} = 0$ then the test is inconclusive.

Example: Find and classify all critical points of the function $f(x, y) = 4 + x^3 + y^3 - 3xy$.

Example: Find the shortest distance from the point $(1,0,-2)$ to the plane $x + 2y + z = 4$.

Absolute Maximum and Minimum Values:

Recall that in one variable this was tantamount to comparing the critical points and end points (which for a planar curve, is the boundary) and seeing which values "won". There is a similar concept in two variables.

Strategy: To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

1. Find the values of f at the critical points of f in D.
2. Find the extreme values of f on the boundary of D.
3. The largest of the values from 1. and 2. is the absolute maximum and the smallest is the absolute minimum.

Example: Find the absolute maximum and minimum values of $f(x, y) = x^2 + 4y^2 - 2x^2y + 4$ on the rectangle given by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

Homework:

- Section 14.7: 5-21, 31-37, 41-53 odds