14.2: Limits and Continuity in Several Variables.

OBJECTIVE

- Introduce limits and continuity in several variables.

In this section we will focus on limits and continuity in two variables, although the same reasoning can be extended to more variables.

In one variable, recall that

$$L = \lim_{x \to a} f(x)$$

if the right and left limit agree. That is, if

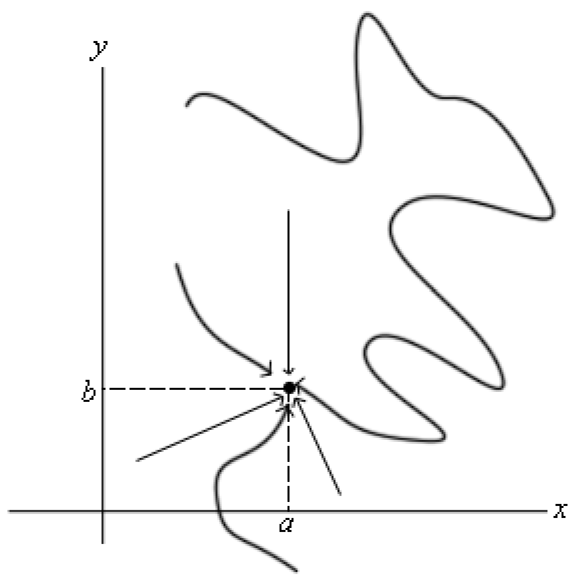
$$L = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

where the RHL requires us to look at value greater than a and the LHL requires us to look at values less than a. (i.e. moving toward a from both sides). We only have *two* approaches.

In two variables, we ask the same question. However, now we take the limit as $x \to a$ and $y \to b$ to find

$$L = \lim_{(x,y)\to(a,b)} f(x,y)$$

In one variable, we said that the limit exists if it approaches the same value from any direction (which only had two directions, the left or right). In two variables, again, the limit exists if it approaches the same value from any direction, but now we have an issue. There are an **infinite** number of approaches to the point (a, b)!



Definition: Let f be a function of two variables whose domain D includes points arbitrarily close to (a, b). Then the limit of $f(x, y)$ as (x, y) approaches (a, b) is L and we write

$$L = \lim_{(x,y)\to(a,b)} f(x, y)$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that if $(x, y) \in D$ and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ then $|f(x, y) - L| < \varepsilon$.

When a limit *does not* exist:

The above definition tells us that the direction of approach shouldn't matter if the limit exists. Thus, if we can find two different approaches that gives two different limit values, then our limit does not exist.

Example: Determine if the following limit exists:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + 3y^4}$$

Continuity:

The definition of continuity in two variables are shown to be defined in a similar way as in one variable.

> **Definition:** A function f of two variables is called continuous at (a, b) if
>
> $$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$
>
> The function f is continuous on D if f is continuous at every point (a, b) in D.

If a function is continuous is continuous at a point (a, b) then to find the limit, simply plug the point into your function and compute.

Fact: Polynomials in two variables are continuous everywhere.

Example: Determine the set of points at which the function is continuous:

$$F(x, y) = \ln(2 + y - x)$$

Homework:

- Section 14.2: 1-21, 29-41 odds