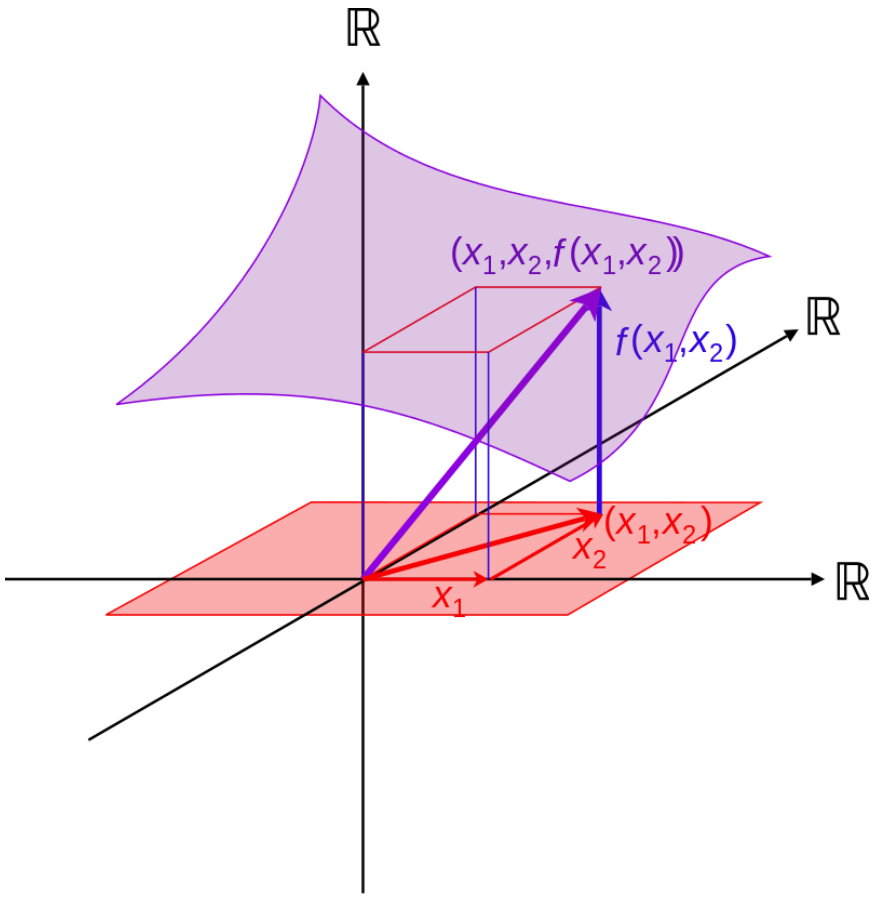
14.1: Functions of Several Variables

OBJECTIVE

- Discuss functions of several variables.
- Introduce graphs of several variables and level curves.

Many applications and models of real-life situations, along with the study of pure mathematics, requires the use of several variables. In this section, we will focus on functions of *two variables*. Consider the elevation of the point on a mountain (as a function of latitude and longitude). Physics tells us that the pressure in a balloon filled with gas is a function of temperature and volume. We will primarily consider functions that input two variables and output one.

Definition: A function of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, i.e. $\{f(x, y) \mid (x, y) \in D\}$.



We write $z = f(x, y)$ to denote our function where x and y are our **independent variables** and z is our **dependent variable**. The domain is understood to be the set of pairs (x, y) for which the expression gives a well-defined real number.

Example: Sketch the graph of $f(x, y) = 12 - 3x - 4y$

Example: Determine the domain of $f(x, y) = \ln(9 - x^2 - 9y^2)$.

Example: Sketch the graph of $f(y,z) = \sqrt{16 - y^2 - z^2}$.

Definition: The **level curves** of a function *f* of two variables are the curves with equations $f(x, y) = k$, where k is a constant in the range of f.

Example: Identify the level curves of $f(x, y) = \sqrt{x^2 + y^2}$. Sketch a few of them.

Functions of three variables:

Example: Find the *level surfaces* of the function

$$f(x, y, z) = x^2 + y^2 + z^2$$

Homework:

- Section 14.1: 9-53, 61-71 odds