# 13.4: Motion in Space

***OBJECTIVE***

- Apply previous results to velocity and acceleration.
- Projectile motion.

For motion in space, we denote the position vector of a particle at time $t$ by $\boldsymbol{r}(t)$. Then we may describe its **velocity vector** by

$$\mathbf{v}(t) = \boldsymbol{r}'(t)$$

which represents the tangent vector pointing in the direction of the tangent line. While velocity is a vector, recall that **speed** is the magnitude of the velocity vector and thus we have

$$\frac{ds}{dt} = |\mathbf{v}(t)| = |\boldsymbol{r}'(t)|.$$

Likewise, the **acceleration** of the particle is given by

$$\mathbf{a}(t) = \mathbf{v}(t) = \boldsymbol{r}''(t).$$

**Example:** Find velocity, acceleration, and speed of a particle with the given position function

$$\boldsymbol{r}(t) = \langle t^2, 2t, \ln t \rangle.$$

**Example:** Find velocity and position vectors of a particle that has the given acceleration and initial velocity and position.

$$\boldsymbol{r}(t) = \langle \sin t\,, 2\cos t\,, 6t \rangle \qquad \mathbf{v}(0) = 3\mathbf{i} - \mathbf{j} \qquad \mathbf{r}(0) = \mathbf{j} + \mathbf{k}.$$

**Projectile Motion:**

A projectile fired with angle of elevation $\alpha$ and initial velocity $\mathbf{v}_0$ has parametric equation given by

$$x = (v_0 \cos \alpha)t \qquad y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$$

where $v_0 = |\mathbf{v}_0|$.

Derive this using Newton's second law of motion, $\mathbf{F} = m\mathbf{a}(t)$.

**Tangential and Normal Components of Acceleration:**

When studying motion, we often break up the acceleration into two components the **tangential** and **normal** components. The tangential component is the part that is tangential to the curve and the normal component is the part that is normal to the curve. With this partition we may write

$$\mathbf{a} = v'\mathbf{T} + \kappa v^2\mathbf{N} = a_{\mathrm{T}}\mathbf{T} + a_{\mathrm{N}}\mathbf{N}$$

where $v = |\mathbf{v}|$ and $\kappa = \frac{|\boldsymbol{r}`(t)\times\boldsymbol{r}``(t)|}{|\boldsymbol{r}`(t)|^3}$.

**Example:** Find the normal and tangential components of the function.

$$\boldsymbol{r}(t) = \langle t, 2e^t, e^{2t}\rangle$$

**Homework:**

- Section 13.4: 3-15, 19-45 odds