13.1 Vector Functions and Space Curves:

OBJECTIVE

- Introduce Vector Functions and Space Curves.

Just as a real-valued function (the ones you are most likely familiar with) take real numbers as their domain and range, vector functions take real numbers as their domain and take vectors as their range. That is, they turn real numbers into vectors. We will be primarily concerned with 3D vectors as our output.

Thus for every real number *t* in the domain, there is a unique vector, denoted by $\vec{r}(t)$ where $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ for real valued functions *f, g,* and *h*, called the component functions. We can write

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}\,.$$

Example: Determine the domain of $\vec{r}(t) = \langle \cos(t), \ln(4-t), \sqrt{t+1} \rangle$.

Limits and Continuity:

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ then

$$\lim_{t\to a} \vec{r}(t) = \langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \rangle$$

provided the limits of the component functions exist.

Example: Find $\lim_{t\to 0} \vec{r}(t)$ where $\vec{r}(t) = \langle \frac{\sin t}{t}, \frac{t^3+2t^2+t}{t}, \cos 4t \rangle$

Note that a vector function $\vec{r}(t)$ is continuous at a point a of

$$\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$$

which requires each component function to be continuous at a.

Space Curves: To graph space curves, we recognize that any vector function can be broken down into a set of parametric equations that represent the same graph. Thus a two dimensional vector function $\vec{r}(t) = \langle f(t), g(t) \rangle$ can be broken down into the parametric equations

$$x = f(t) \quad \text{and} \quad y = g(t)$$

And a 3D vector function $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ can be broken down into the parametric equations

$$x = f(t) \quad \text{and} \quad y = g(t) \quad \text{and} \quad z = h(t).$$

Example: Sketch the graph of the following vector function

$$\vec{r}(t) = \langle 2 - 4t, -1 + 5t, 3 + t \rangle.$$

Example: Sketch the graph of the following vector function

$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle.$$

Example: Find the vector equation and parametric equations for the line segment that joins P(-1,2,-2) and Q(-3,5,1).

Example: Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

Homework:

- Section 13.1: 1-31 & 41-45 odds